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DURATION OF THE FREEZING OF BODIES WITH VARIABLE TEMPERATURE  
OF THE MEDIUM

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The article contains an analysis of the application of the integral method of thermal moments of zeroth order in determining the duration of freezing of bodies with simple shape when the temperatures of the cooling medium is variable.

Approximate analytical solutions of unidimensional Stefan-type problems for determining the duration of processes of nonsteady heat conduction are conveniently found by using the so-called integral methods [1]. To these also belongs the method of thermal moments of zeroth order [2]. The application of this method to problems with phase transformations at constant temperature of the medium was studied, e.g., in [3, 4]. The essence of the method is that the initial integral relation is obtained as a result of integrating the principal differential equation of heat conduction twice with respect to the space coordinate and once with respect to time. Into this relation we then substitute the equations of the temperature distribution profiles (invariant to shifts of the front of phase transformation) and the regularity of change of the cooling (heating) impulse on the surface of the body, determined as the area in coordinates temperature vs time between the lines of temperature change at the end of the investigated region (body).

The method of thermal moments of zeroth order may also be applied to determining the time of motion of the fronts of phase transformation in bodies of simple shape when the temperature of the medium is variable. Although it is expedient to use the integral statement of the problem [4], we demonstrate below how to obtain the initial integral relation of the thermal moments from the differential statement of the problem because the method itself is relatively little known.

Let us examine the problem of the cooling of bodies with simple shape (sphere, unbounded cylinder, and plate) with phase transformations

$$C(T)\omega(x)\frac{\partial T(x,\tau)}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda(T)\omega(x)\frac{\partial T(x,\tau)}{\partial x}\right), \quad 0 \leq x \leq l; \quad (1)$$

$$T(x, 0) = T_0(x); \quad (2)$$

$$\frac{\partial T(0, \tau)}{\partial x} = 0; \quad (3)$$

$$\alpha(\tau)(T(l, \tau) - T_c(\tau)) = -\lambda(T(l, \tau))\frac{\partial T(l, \tau)}{\partial x}; \quad (4)$$

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$$\frac{\partial T(x, \tau)}{\partial x} \leq 0, \quad 0 \leq x \leq l, \quad \tau \geq 0. \quad (5)$$

To fulfill condition (5) that the direction of the heat flux be constant, it suffices that  $dT_c/d\tau \leq 0$  for  $\tau \geq 0$ ,  $dT_0/dx \leq 0$ .

After the first integration with respect to  $x$  we obtain at some instant  $\tau$ , taking (3) into account, that

$$\frac{\partial T(x, \tau)}{\partial x} = \frac{1}{\lambda(T)\omega(x)} \int_0^x C(T)\omega(\xi) \frac{\partial T(\xi, \tau)}{\partial \tau} d\xi. \quad (6)$$

If we integrate both sides of (6) with respect to  $x$  once more, we find

$$T(x, \tau) = \int_0^x \frac{dy}{\lambda(T)\omega(y)} \int_0^y C(T)\omega(\xi) \frac{\partial T(\xi, \tau)}{\partial \tau} d\xi + C_2(\tau). \quad (7)$$

Equation (7) may be written in the form of a double integral

$$T(x, \tau) = \iint_{(S)} \frac{\omega(\xi)C(T(\xi, \tau))}{\omega(y)\lambda(T(y, \tau))} \frac{\partial T(\xi, \tau)}{\partial \tau} dyd\xi + C_2(\tau)$$

with respect to the region  $S[0 \leq y \leq x; 0 \leq \xi \leq y]$ . This double integral is a different form of the multiple integral

$$T(x, \tau) = \int_0^x C(T)\omega(\xi) \frac{\partial T}{\partial \tau} \left( \int_{\xi}^x \frac{dy}{\lambda(T)\omega(y)} \right) d\xi + C_2. \quad (8)$$

After having determined  $C_2$  from condition (4), we obtain finally for any  $x \in [0, l]$ :

$$T(x, \tau) - T_c(\tau) = \int_0^x C(T)\omega(\xi) \frac{\partial T}{\partial \tau} \left( \int_{\xi}^x \frac{dy}{\lambda(T)\omega(y)} \right) d\xi - \int_0^l C(T)\omega(\xi) \frac{\partial T}{\partial \tau} \left( \frac{1}{\alpha(\tau)\omega(l)} + \int_{\xi}^l \frac{dy}{\lambda(T)\omega(y)} \right) d\xi, \quad (9)$$

and hence for the center of the body ( $x = 0$ ) at any instant  $\tau$ :

$$T(0, \tau) - T_c(\tau) = - \int_0^l C(T)\omega(x) \frac{\partial T}{\partial \tau} \left( \frac{1}{\alpha(\tau)\omega(l)} + \int_x^l \frac{dy}{\lambda(T)\omega(y)} \right) dx. \quad (10)$$

Thus one integral equation (9) with the initial condition (2) corresponds to the system of differential equations (1)-(5).

We integrate both sides of Eq. (10) with respect to time:

$$\Omega \equiv \int_0^{\tau_f} (T(0, \tau) - T_c(\tau)) d\tau = - \int_0^{\tau_f} d\tau \int_0^l C(T)\omega(x) \frac{\partial T}{\partial \tau} \left( \frac{1}{\alpha(\tau)\omega(l)} + \int_x^l \frac{dy}{\lambda(T)\omega(y)} \right) dx \equiv \Delta H. \quad (11)$$

The geometric interpretation of the left-hand side of Eq. (11) with condition (5) is obvious: it is the area in coordinates  $T-\tau$  between the lines of change of temperature  $T(0, \tau)$  and  $T_c(\tau)$  within the time  $\tau_f$ . In view of the fact that the function  $T \equiv T(x, \tau)$  is continuous in the region of its determination, the order of integration on the right-hand side of (11) may be changed:

$$\Delta H \equiv \int_0^l \omega(x) \left( - \int_0^{\tau_f} C(T) \left( \frac{1}{\alpha(\tau)\omega(l)} + \int_x^l \frac{dy}{\lambda(T)\omega(y)} \right) \frac{\partial T}{\partial \tau} d\tau \right) dx.$$

The expression in the outer parentheses is the integral depending on the parameter  $x$ . With any fixed  $x = x_*$  this integral is obtained by the substitution of  $T(x_*, \tau) \equiv T_*(\tau)$  for  $\tau$  from the integral

$$\Phi(x_*) = \int_{T_*(\tau_f)}^{T_*(0)} C(T_*) \left( \frac{1}{\alpha(T_*, x_*) \omega(l)} + \int_{x_*}^l \frac{dy}{\lambda(T_*, x_*, y) \omega(y)} \right) \delta T, \quad (12)$$

where  $T_*(0) = T_0(x_*)$ .

Expression (12) is correct for any  $0 \leq x \leq l$ , it therefore determines the parametric specification of the function, and the parameter  $x$  is contained in the integrand as well as in the integration limits. Consequently, the following function is determined:

$$\Phi(x) = \int_{T(x, \tau_f)}^{T_0(x)} C(T) I(T, x) \delta T, \quad (13)$$

where

$$I(T, x) = \frac{1}{\alpha(T, x) \omega(l)} + \int_x^l \frac{dy}{\lambda(T, x, y) \omega(y)},$$

and relation (11) may be written as follows:

$$\Omega \equiv \int_0^{\tau_f} (T(0, \tau) - T_c(\tau)) d\tau = \int_0^l \omega(x) \int_{T(x, \tau_f)}^{T_0(x)} C(T) I(T, x) \delta T dx \equiv \Delta H. \quad (14)$$

Thus we integrated (1) twice with respect to  $x$  and once with respect to  $\tau$ , taking all the boundary conditions into account. We obtained the integral relation (14) which can be resolved relative to the duration  $\tau_f$  if we know the regularity of the change of area of the configuration  $\Omega(\tau_f)$ . On the other hand, like with all integral methods, we have to specify the temperature distribution profiles across the thickness of the body  $t = t(T, x, y)$  which are substituted into the functions  $\alpha(\tau) = \alpha(T(y, \tau))$  and  $\lambda(T(y, \tau))$  instead of  $T$  for the purpose of obtaining the dependences  $\alpha(T, x)$  and  $\lambda(T, x, y)$ .

For greater lucidity the integral relation (14) upon its construction may be represented by Stieltje's double integral over the region  $\sigma[0 \leq x \leq l; T(x, \tau_f) \leq T < T_0(x)]$ :

$$\Omega \equiv \int_0^{\tau_f} (T(0, \tau) - T_c(\tau)) d\tau = \iint_{(\sigma)} I(T, x) d^2 Q(T, x) \equiv \Delta H. \quad (15)$$

Relation (15) as the integral form of Fourier's law of heat conduction [4] has an analogy with the law of conservation of momentum in mechanics:  $\Omega$  is the integral impulse of force whose analog is the temperature difference, and  $\Delta H$  is the "momentum" of heat against the thermal resistances or the moment of the amount of heat of zeroth order. In fact, the integrand in the double integral is the product of the amount of heat  $d^2 Q = C(T) \omega(x) \delta T dx$ , released from an elementary layer with volume  $\omega(x) dx$  when the temperature in it is lowered by  $\delta T$ , and the instantaneous thermal resistance  $I(T, x)$  from this layer to the cooling medium at the instant when the mentioned lowering of the temperature occurs.

The thermal impulse  $\Omega$  expresses the measure of the thermal effect on the body, and  $\Delta H$  is the change of the thermal state of the body under this effect. Depending on the direction of the heat flux, the impulse  $\Omega$  may be taken to be positive or negative. From the thermodynamic point of view  $\Omega < 0$  and  $\Delta H < 0$  upon cooling of bodies. But since their signs always coincide, it is expedient in thermophysical calculations to take the temperature difference at  $\Omega$  as positive, and correspondingly to assume that  $\Delta H > 0$ .

Using the above-mentioned analogies, we can easily construct the integral form of stating the problem in the form of relation (15). It expresses the theorem of additivity of the magnitude  $\Delta H$ . To each element of the area of the region  $d^2 \sigma = \delta T dx$  corresponds an elementary amount of heat released into the environment  $d^2 Q$  whose "momentum" against the resistance  $I$  into the environment is determined as  $d^2 H = I d^2 Q$ . Here,  $d^2 H$  is always correlated with the corresponding increase of the thermal impulse of the effect, i.e.,

$$d^2 \Omega \equiv \delta(T(0, \tau) - T_c(\tau)) d\tau = I(T, x) d^2 Q(T, x) \equiv d^2 H. \quad (16)$$

If  $\lambda = \text{invar}$  or  $\lambda = \lambda(x)$  and  $\alpha = \text{invar}$ , the thermal resistance  $I = I(x)$  and relation (15) are greatly simplified:

$$\Omega \equiv \int_0^{\tau_f} (T(0, \tau) - T_c(\tau)) d\tau = \int_0^l I(x) dQ(x) \equiv \Delta H, \quad (17)$$

where

$$dQ(x) = \left( \int_{T(x, \tau_f)}^{T_0(x)} C(T) \delta T \right) \omega(x) dx.$$

The integral relation (15) may be used for determining the length  $\tau_f$  of the process of cooling of the body. The approximation of its left-hand side is possible if we know the change of temperature of the center of the body  $T(0, \tau)$ . This is particularly convenient in problems with phase transformations. Then, with the nature of the change  $T_c(\tau)$  specified, we can determine the time dependence of the change of the area  $\Omega$  between the lines of change of these temperatures in coordinates  $T-\tau$ . In other cases, when the problem is stated so that  $\tau_f$  can be determined, it is more convenient to use relation (15) in the form of the first differentials of  $\Omega$  and  $H$  with subsequent integration. In this case the time  $\tau$  is treated as the sought function, and the temperature as the independent coordinate since the temperature distribution profiles are being specified.

The momentum of the heat  $\Delta H$  is an additive magnitude only over the region  $\sigma$ , i.e., along the path of the heat flux. Along the surface  $\omega(l)$  the magnitude  $\Delta H$  is not additive. From the thermodynamic point of view  $H$  becomes a singular function of state in the case of the equilibrium process of cooling of the body, when  $Bi \rightarrow 0$ . With arbitrary dependences  $C(T)$ ,  $\lambda(T)$ , and  $\alpha(T)$ , the value of  $\Delta H$  depends only on the initial temperature ( $T_0$ ) and the final temperature ( $T_f$ ) of the body

$$\Delta \hat{H} = \hat{H}(T_0) - \hat{H}(T_f) = \int_0^l \int_{T_f}^{T_0} \frac{C(T)}{\lambda(T)} \omega(x) \int_x^l \frac{dy}{\omega(y)} dx \delta T. \quad (18)$$

The property of additiveness of relation (15) over the region  $\sigma$  enables us to go over easily from the nonclassical statement of the problem of the freezing of bodies to the classical statement with the Stefan condition. In this case the expression for  $\Delta H$  contains additionally the component of the heat of phase transformation

$$\Delta H_p = \int_V^0 Q_p \omega(\xi) I(T, \xi) d\xi. \quad (19)$$

Using the theorem of the additiveness of the impulse, "the momentum" of heat, i.e., relations (15)-(17), we can approximately solve the problem of the duration of the processes of nonsteady heat conduction, neglecting any of the factors of thermal influence and estimating the error due to such a simplification according to the possible deviation of the function  $\Delta H$ . It is expedient to use the known theorems of comparison [5]. If  $C$ ,  $\lambda$ ,  $\alpha$  do not depend on  $T$  and  $\tau$ , then in resolving the integral relation (15) we need the temperature distribution profile with respect to  $x$  to be specified only at the final instant  $\tau_f$ , otherwise the approximation of the temperature profiles for any  $\tau$  is indispensable.

In order to illustrate the method of solving the problem of the freezing of bodies by using relation (15), we will examine the plane unidimensional single-phase problem of Stefan in the classical statement with boundary condition of the first kind, when the front of the phase transformation moves into the interior of the space at the constant speed  $v = \alpha K$ . In that case the exact solution of Stefan [6] for  $x = 0$  on the surface of the body is known

$$T_p - T(x, \tau) = \frac{Q_p}{C} \left( \exp \left( \frac{v^2 \tau - vx}{a} \right) - 1 \right). \quad (20)$$

Then, having  $T(x, 0) = T_p$ , we find in the layer with thickness  $dx$  during the entire time  $\tau$ :

$$dH = (C(T_p - T(x, \tau)) + Q_p) \frac{x}{\lambda} dx. \quad (21)$$

Integration of (21) from  $x = 0$  to  $\xi = v\tau$ , taking (20) into account, yields

$$\Delta H = \frac{Q_p}{C} \left( \frac{a}{v^2} \left( \exp \left( \frac{v^2 \tau}{a} \right) - 1 \right) - \tau \right). \quad (22)$$

on the other hand, in accordance with (20):

$$d\Omega = \frac{Q_p}{C} \left( \exp \left( \frac{v^2 \tau}{a} \right) - 1 \right) d\tau,$$

whose integration leads to an expression that is identical with (22). Thus we verified that the integral relation (15) is exact.

We will demonstrate the application of the method to the approximate solution of the problem of the freezing of bodies with simple shape stated as follows:

$$\omega(x) \frac{\partial T}{\partial \tau} = a \frac{\partial}{\partial x} \left( \omega(x) \frac{\partial T}{\partial x} \right), \quad 0 \leq x \leq l; \quad (23a)$$

$$T(x, 0) = T_p = T(0, \tau); \quad (23b)$$

$$-\lambda \frac{\partial T(l, \tau)}{\partial x} = \alpha(T_s(\tau) - T_c(\tau)); \quad (23c)$$

$$-\lambda \frac{\partial T(\xi, \tau)}{\partial x} = Q_p \frac{d\xi}{d\tau}, \quad (23d)$$

when the temperature of the medium decreases at constant speed  $b = -dT_c/d\tau > 0$  according to the regularity  $T_c(\tau) = T_{co} - b\tau$ ;  $T_{co} \equiv T_c(0) \leq T_p$ . Then for any  $\tau$  we have  $\Omega = (T_p - T_{co}) \cdot \tau + b/2 \tau^2$ . We specify that the temperature profile at the end of the freezing is linear

$$T(x) = T_p - (T_p - T_c) \frac{\alpha x}{\alpha l + \lambda}. \quad (24)$$

Determining

$$I(x) = \frac{1}{\alpha \omega(l)} + \frac{1}{\lambda} \int_x^l \frac{dy}{\omega(y)}$$

and  $dQ(x) = (C(T_p - T(x)) + Q_p)\omega(x)dx$ , we find

$$\Delta H = \int_0^l I(x) dQ(x) = \frac{Q_p l^2 (2 + Bi)}{2\lambda f Bi} + \frac{l^2 (T_p - T_c) (3 + Bi)}{3a(f+1)(1+Bi)}. \quad (25)$$

Then from the relation  $\Omega = \Delta H$  we obtain finally the formula of the duration of complete freezing of bodies with simple shape

$$\tau_0 = 2E(F + \sqrt{F^2 + 2bE})^{-1}, \quad (26)$$

where

$$F = T_p - T_{co} - bD; \\ E = \frac{Q_p l^2 (2 + Bi)}{2f\lambda Bi} + D(T_p - T_{co}); \quad D = \frac{l^2 (3 + Bi)}{3a(f+1)(1+Bi)}.$$

With  $b = 0$ , the known formula of the time of freezing of bodies with constant  $T_c$  follows from (26) [3].

The effect of  $Q_p$  in (26) was taken accurately into account, therefore the error of this formula depends only on the error in determining the "momentum" of the heat due to the heat capacity of the frozen zone, when the profile  $T(x)$  is specified for the end of the freezing.

In the case of the Rudolf Planck problem, when instead of the system of equations (23) only Eq. (23d) is solved, i.e.,  $C \rightarrow 0$ , the following exact relation follows from (26):

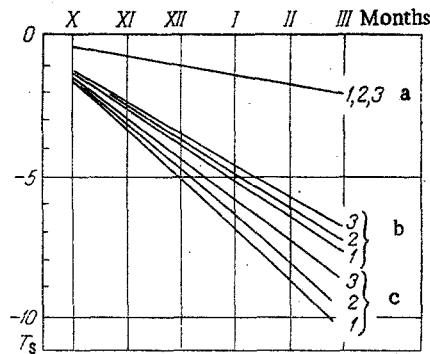


Fig. 1. Results of the calculation of the soil surface temperature in the winter period with constant speed of the freezing front: 1) exact Stefan solution of (20); 2) by formula (28); 3) by formula (27); a) moist loam; b) sandy loam; c) sand.  $T_s$ , °C.

$$b = \frac{Q_p(2 + Bi)}{\lambda f Bi} v^2, \quad (27)$$

establishing a quadratic correlation between the simultaneously existing constant speeds  $b$  and  $v$ . However, when  $C$  is taken into account, the simultaneous constancy of these speeds does not apply any more. Thus the equation of the correlation of speeds (27) applies only for large  $Ko$ .

To verify the accuracy of formula (26) we will examine a numerical example from physical geocryology. It is known that the front of the seasonal freezing of the soil, having a mean annual temperature close to  $T_p = 0^\circ\text{C}$ , moves in the course of almost six winter months at the practically constant speed  $v$ . We determine the change of the soil surface temperature directly under the snow cover by differentiating (25) with respect to  $\tau$  for  $\tau = \tau_f$ ,  $Bi \rightarrow \infty$ ,  $f = 1$ , and taking into account that  $l = -v\tau$  and  $d(\Delta H)/d\tau = d\Omega/d\tau = T_p - T_s$ . Then

$$T_s = T_p - \frac{3Q_p v^2 \tau}{3\lambda - C v^2 \tau}. \quad (28)$$

Figure 1 presents the results of the comparative calculation of the change of the soil surface temperature  $T_s$  by formulas (20), (27), and (28) for three types of soil: a) loam with  $v = 1.5 \cdot 10^{-4}$  m/h; b) sandy loam with  $v = 3.5 \cdot 10^{-4}$ ; c) sand with  $v = 5 \cdot 10^{-4}$ . The value of  $Ko$  for  $\tau_f = 6$  months was 26.0, 5.6, and 3.1, respectively. In the case (a) with large values of  $Ko$ , the three solutions practically coincide. Formula (27), obtained for the linear profile  $T(x)$ , yields the largest divergence compared with the exact solution of (20) in the case (c): 5%.

#### NOTATION

$x$ , linear space coordinate with the origin at the center of the sphere, of the unbounded cylinder or plate;  $\omega(x) = 1, 2\pi x, 4\pi x^2$ , for the plate, cylinder, and sphere, respectively;  $f = 1, 2, 3$ , shape factor of these bodies;  $l$ , half-thickness for the plate and radius for the cylinder and sphere;  $\xi$  and  $v = d\xi/d\tau$ , depth and speed of freezing of the body, respectively;  $\tau, \tau_f$ , duration of the cooling of the body;  $T$ , temperature;  $T_c$ , temperature of the medium;  $T_s$ , temperature on the surface of the body;  $T_p$ , temperature of the phase transformation;  $b$ , rate of the temperature decrease of the medium;  $C$  and  $\lambda$ , volumetric heat capacity and thermal conductivity, respectively, of the substance of the body in the frozen state;  $Q_p$ , volumetric heat of the phase transformation;  $\alpha$ , heat-transfer coefficient on the surface of the body;  $Bi \equiv \alpha l / \lambda$ ;  $Ko \equiv Q_p / C(T_p - T_c)$ , Kossovich number.

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#### LIQUID BOILING PROCESS IN ROTATING VESSELS AND CHANNELS

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Results of visual observations of water boiling in rotating vessels and channels are presented. The existence of various forms of nucleate boiling is established. Simplified calculations of single phase and boiling liquid thermal convection are performed.

In order to construct high-power electric generators with rotors cooled by cryogenic liquids [1], information is required on the motion of heated and boiling liquid in rotating cavities and channels of various configurations. Reviews of studies of the boiling process in rotating vessels can be found in [2-5]. Visual observations of liquid flow in such vessels were described in [6] and other studies. In view of the shortcomings and contradictions of the available studies, the present authors carried out test stand studies in which the convection and boiling of heated water in rotating glass vessels and channels of various form were observed (Fig. 1). Test stand parameters were as follows: radial distance from bottom of vessel or channel wall to axis of rotation  $r_2 \approx 12$  cm, radial distance from edge of vessel or beginning of channel to axis of rotation  $r_1 = 2$  cm, channel diameter  $D = 0.3$  or  $0.5$  cm, vessel diameter  $4$  or  $7$  cm, heater, externally heated sleeve around channel section  $L_2 = 8$  cm long, or  $3$  cm diameter plane heater immersed to bottom of vessel, occupying from  $18$  to  $56\%$  of bottom area. Experiments were performed both with and without transparent screens to protect the rotating vessels and channels from cooling by the air through which they moved. The screenless experiments produced additional heat losses, but simplified observations. Angular rotation frequency  $\omega \leq 157 \text{ sec}^{-1}$  (or  $\leq 1500$  rpm). This corresponded to a maximum centripetal acceleration of  $r_2\omega^2 \leq 3 \cdot 10^3 \text{ m/sec}^2$  or relative acceleration of  $G = r_2\omega^2 g^{-1} \leq 300$ , and an excess pressure produced by centrifugal forces of  $P_+ = 0.5\rho\omega^2 \times (r_2^2 - r_1^2) \leq 0.17 \text{ MPa}$ , with increase in the water boiling point at the bottom of the vessel to  $130^\circ\text{C}$ .

Visual observations and photography of the motion of vapor bubbles and plastic shavings with a density close to that of water were carried out under stroboscopic illumination. The rotation frequency and thermal heating power  $Q$  were measured.

Boiling in a free volume was studied in a glass vessel  $9$  cm high with axis oriented along the normal to the axis of rotation (8, Fig. 1), with water supplied through collector 4.

Observations of the convective motion of the single phase liquid not heated to the boiling point ( $Q \leq 50 \text{ W}$ ) were performed under normal conditions  $G > 30$  and  $Ra_\omega = D^3\omega^2 r_2 \beta T_+ (\nu\alpha)^{-1} \approx 10^8 - 10^{10}$ . In this case it was evident from the motion of the suspended particles that the usual (in the absence of rotation) two-loop convection with a flow of hot liquid departing from the center of the heater did not occur. Instead, a one-loop circulation convection was established with cold flow directed along the pressure wall of the vessel to the bottom and heater located there, with heated flow directed along the pressure wall of the vessel to the bottom and heater located there, with heated flow directed from the heater along the nonpressure wall toward the axis of rotation (1, Fig. 2). The direction of the

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